IIT JEE 2009 Test Series 4
MATHEMATICS PART-II

SECTION I
STRAIGHT OBJECTIVE TYPE
This section contains 9 multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which ONLY ONE is correct. 3 MARKS will be awarded for correct answer. 1 MARK will be deducted for wrong answer.

Q1. A man has 10 friends among whom two are married to each other. Then the number of different ways in which he can invite 5 people to a dinner party if married people refuse to attend separately is

(A) ${}^{10}C_5 - 2$
(B) ${}^{10}C_5 - 4*{}^{8}C_4$
(C) $2*{}^{8}C_3$
(D) 224

Q2. $f(x) = f(x)f(y)$, $f(x)$ is continuous at x=a. Then,

(A) $f(x)$ is continuous everywhere
(B) $f(x)$ is continuous everywhere except at x= -1,0,1
(C) $f(x)$ is discontinuous everywhere except at x=a
(D) $f(x)$ is continuous everywhere except at x=0

Q3. The domain of $f(x) = \sqrt{x^{14} - x^{11} + x^6 - x^3 + x^2 + 1}$ is

(A) [0,∞)
(B) (-∞, 0]
(C) [-1, 1]
(D) R

Q4. The sides of a triangle are the roots of the cubic equation $x^3 - 13x^2 + 54x - 72 = 0$ then the area of the triangle in square units is

(A) $\frac{1}{2}\sqrt{455}$
(B) $\frac{1}{4}\sqrt{455}$
(C) $\frac{1}{2}\sqrt{394}$
(D) $\frac{1}{4}\sqrt{394}$

Q5. If a, b, c are in H.P. and p is the length of the perpendicular drawn from the origin to any member of family of lines (a+c-2ac)bx-2acy+7abc=0 then p ϵ

(A) $[\frac{1}{4}, \frac{9}{\sqrt{2}}]$
(B) $[0, \frac{7}{\sqrt{2}}]$
(C) (7,9)
(D) $[\frac{7}{\sqrt{2}}, \frac{9}{\sqrt{2}}]$

Q6. If $\frac{\sin(xy)-2\tan(xy)+4cosec(xy)}{\cos(xy)+4\sec(xy)-2\cot(xy)} = 14$ then $\frac{dx}{dy}$ at (4, 2) is

(A) 0
(B) 4
(C) -2
(D) 2

Q7. If $f(n) = \int_0^{\pi/2} (\frac{\sin nx}{sinx})^2 dx$ then

(A) $f(1) = \pi$
(B) $\sin f(7) + cosf(7) = -1$
(C) $\sin f(7) + cosf(7) = 1$
(D) $\tan^2 f(2) + cot^2 f(2) = 4$

Q8. Let $S(n)$ denotes the sum of first n terms of an A.P. If $f(n) = \frac{S(3n)}{s(2n)-S(n)}$ then $S = \lim_{n\to\infty} \sum_{r=-n}^{n} \frac{f(r)}{n}$ equals

(A) 6
(B) 8
(C) 12
(D) 16

Q9. The area enclosed by the image of $x^2 - y^2 = 1$ under inversion (ω=1/z)in Argand plane(ω= u + *i*v) is

(A) 1/2
(B) (B) 1
(C) (C) 2/3
(D) (D) 1/4

SECTION II
ASSERTION-REASON TYPE
This question contains 4 reasoning type questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct. 3 MARKS will be awarded for correct answer. 1 MARK will be deducted for wrong answer. NO MARKS WILL BE GIVEN OR DEDUCTED IF A QUESTION IS NOT ANSWERED.

Q10. Let $f(x) = 2 + \cos x$ for all real x.

STATEMENT-1: For each real t, there exists a point c in $[t, t + \pi]$ such that $f'(c) = 0$.

STATEMENT-2: $f(t) = f(t + 2\pi)$ for each real t.

(A) STATEMENT-1 is True, STATEMENT-2 IS True; STATEMENT -2 is a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 IS True; STATEMENT -2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 IS False
(D) STATEMENT-1 is False, STATEMENT-2 IS True

Q11. Consider the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.
STATEMENT-1: The parametric equations of the line of intersection of the given planes are $x = 3 + 14t$, $y = 1 + 2t$, $z = 15t$

STATEMENT-2: The vectors $14i + 2j + 15k$ is parallel to the line of intersection of the given planes.

(A) STATEMENT-1 is True, STATEMENT-2 IS True; STATEMENT -2 is a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 IS True; STATEMENT -2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 IS False
(D) STATEMENT-1 is False, STATEMENT-2 IS True

Q12. **STATEMENT-1:** If a, b, c, are positive real numbers, then $[(1 + a)(1 + b)(1 + c)]7 > 7^7 a^4 b^4 c^4$

.

STATEMENT-2: $(1 + a)(1 + b)(1 + c) = 1 + ab + a + b + c + abc + ac + bc$ using AM $\geq$ GM, we get statement 1

(A) STATEMENT-1 is True, STATEMENT-2 IS True; STATEMENT -2 is a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 IS True; STATEMENT -2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 IS False
(D) STATEMENT-1 is False, STATEMENT-2 IS True

Q13. Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and a, $\frac{1}{\beta}$ are the roots of the equation $ax^2 + 2bx + c = 0$, where β^2 does not belongs to {-1, 0, 1}

STATEMENT-1: $(p^2 - q)(b^2 - ac) \geq 0$

STATEMENT-2: $b \neq pa$ or $c \neq qa$

(A) STATEMENT-1 is True, STATEMENT-2 IS True; STATEMENT -2 is a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 IS True; STATEMENT -2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 IS False
(D) STATEMENT-1 is False, STATEMENT-2 IS True

SECTION III
LINKED COMPREHENSION TYPE
This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has four choices (a), (b), (c) and (d), out of which ONLY ONE is correct. 4 MARKS will be awarded for correct answer. 1 MARK will be deducted for wrong answer. NO MARKS WILL BE GIVEN OR DEDUCTED IF A QUESTION IS NOT ANSWERED.

Paragraph for Questions numbers 14 to 16

A biased coin shows head with a probability of ¾ and tail with a probability of ¼. Let p_n denotes the probability that no three or more heads appear consecutively in n throws of the coin.

Q14. If $p_n = \alpha p_{n-1} + \beta p_{n-2} + \gamma p_{n-3}$ then $64(\alpha + \beta + \gamma)$ is equal to

(A) 37 (B) 23 (C) 121 (D) 119

Q15. The value of p_4 is

(A) $\frac{121}{256}$ (B) $\frac{23}{64}$ (C) $\frac{37}{64}$ (D) $\frac{119}{256}$

Q16. If the coin is tossed 4 times and let A is the event that three or more heads occurs in four tosses and B is the event that three heads do not occur in first three tosses, then P(A/B)

(A) $\frac{27}{148}$ (B) $\frac{4}{37}$ (C) $\frac{81}{148}$ (D none of these

Paragraph for Questions numbers 17 to 19

Let A be a set containing *n* elements. A subset P of A chosen and the set A is reconstructed by replacing the elements of P. A subset Q of A is chosen again. Find the number of ways of choosing P and Q.

Q17. If P and Q have equal number of elements

(A) $\sum_{r=0}^{n}\binom{n}{r}^2$
(B) 4^n
(C) $\binom{2n}{n-1}$
(D) none of these

Q18. If Q contains just one element more than P

(A) $\binom{2n}{n-1}$
(B) $\binom{2n}{n}$
(C) $\sum_{r=0}^{n-1}\binom{n}{r}\binom{n}{r-1}$
(D) none of these

Q19. If Q contains atleast one element more than P

(A) 2^{2n-1}
(B) $\binom{2n}{n}$
(C) $2^{2n-1} - \frac{1}{2}\binom{2n}{n}$
(D) $2^{2n-1} + \frac{1}{2}\binom{2n}{n}$

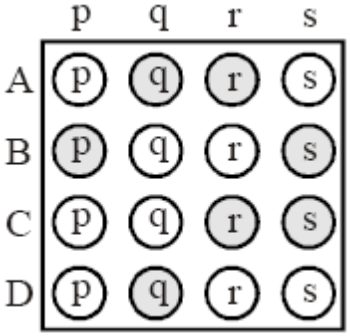
SECTION-IV

Matrix Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements in **Column I** are labelled as A, B, C and D whereas statements in **Column II** are labelled as p, q, r and s. The answers to these questions have to be appropriately bubbled asillustrated in the following example.

If the correct matches are A-q, A-r, B-p, B-s, C-r, C-s and D-q, then the correctly bubbled

matrix will look like the following :



Q20. z1, z2, z3 represents distinct points A, B, C in Argand's plane.

	Column 1		**Column 2**
(a)	If $\alpha \neq 1$, $\alpha^5=1$,then $\log_{\sqrt{3}}[1+\alpha+\alpha^2+\alpha^3-(2/\alpha)]$ =	(p)	3
(b)	A variable chord PQ of $y^2=4x$ passes through the point R(b,0). If $[\frac{1}{PR^2}+\frac{1}{QR^2}]$ is constant then b =	(q)	$2\sqrt{2}$
(c)	The length of the latus-rectum of xy=1 is	(r)	$2\sqrt{3}$
(d)	A variable straight line through the point A (-1, 1) intersects the ellipse $x^2+2y^2=2$ at B and C.P is a point on BC such that AB, AP, AC are in H.P. The least distance of the point (4, 0) from the locus of P is	(s)	2

Q21. If L = $\lim_{x\to 0}\frac{a\sin x - bx + cx^2 + x^3}{2x^2\log(1+x) - 2x^3 + x^4}$ is finite then match the following:

	Column I		**Column II**
(a)	a + b	(p)	0
(b)	c	(q)	6
(c)	20L	(r)	12
(d)	a	(s)	1.5

Q. 22.

	Column I		**Column II**
(a)	The ratio of the altitude to the radius of the cylinder of maximum value that can be inscribed in a given sphere is	(p)	$1/\sqrt{2}$
(b)	The ratio of radius to the altitude of the cone of the greatest volume which can be inscribed in a given sphere is	(q)	$\sqrt{2}$
(c)	The cone circumscribing of sphere of radius r has the minimum volume if its semi vertical angle is α is such that $33\sin\alpha$ =	(r)	32/3
(d)	The greatest value of x^3y^4 if $2x + 3y = 7$ and $x \geq 0$, $y \geq 0$	(s)	11