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IIT JEE 2009 Test Series 3

MATHEMATICS PART-II

SECTION I

STRAIGHT OBJECTIVE TYPE

This section contains 6 multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which ONLY ONE is correct. 3 MARKS will be awarded for correct answer. 1 MARK will be deducted for wrong answer.

1. Let $f(x) = g(x)\frac{e^{\frac{1}{x}}-e^{\frac{-1}{x}}}{e^{\frac{1}{x}}+e^{\frac{-1}{x}}}$ and x≠0, where g is a continuous function Then $\lim_{x\to 0} f(x)$ exists if

(A) *g(x)* is any polynomial
(B) *g(x)* = *x* + 4
(C) *g(x)* = x^2
(D) *g(x)* = 2 + 3*x* + 4x^2

2. If the three planes r.n1 = p1, r.n2 = p2 and r.n3 = p3 have a common line of intersection then
(A) p1(n2xn1)+p2(n3xn1)+p3(n1xn2) = 0
(B) – p1(n2xn1) – p2(n3xn1)+p3(n1xn2) = 0
(C) p1(n2xn1) – p2(n3xn1) – p3(n1xn2) = 0
(D) NONE OF THESE

3. The solution of the differential equation $(xy^4 + y)dx - xdy = 0$ is
(A) $\frac{x^4}{4} + \frac{1}{3}(\frac{x}{y})^3 = C$
(B) $\frac{x^4}{4} - \frac{1}{3}(\frac{x}{y})^3 = C$
(C) $\frac{x^3}{9} + \frac{1}{4}(\frac{x}{y})^4 = C$
(D) $\frac{x^4}{16} + \frac{1}{27}(\frac{x}{y})^3 = C$

4. The value of the integral $\int_0^1 \frac{x^b - 1}{\log x} dx (b > 0)$ is

(A) log b
(B) 2 log(b+1)
(C) 3 log b
(D) NONE

5. $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx =$

(A) $\sin^{-1}(\frac{1}{3}(\sin x + \cos x)) + C$
(B) $\cos^{-1}(\frac{1}{3}(\sin x - \cos x)) + C$
(C) $\sin^{-1}(\frac{1}{3}(\sin x - \cos x)) + C$
(D) $\cos^{-1}(\frac{1}{3}(\sin x + \cos x)) + C$

6. The equation $|z - z_1|^2 + |z - z_2|^2 = k$ (where k is a real number) will represent a circle if
(A) $k \geq \frac{1}{2}|z_1 - z_2|^2$
(B) $k \leq \frac{1}{2}|z_1 - z_2|^2$
(C) $k \leq \frac{1}{2}|z_1 + z_2|^2$
(D) $k \geq \frac{1}{2}|z_1 + z_2|^2$

7. If $f(x) = \begin{cases} (1 + |\sin x|)^{\frac{a}{|\sin x|}}, & -\frac{\pi}{6} < x < 0 \\ b, & x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}}, & 0 < x < \frac{\pi}{6} \end{cases}$

is a continuous function on $(-\frac{\pi}{6}, \frac{\pi}{6})$, then

(A) a = 2/3, b = $e^{1/3}$
(B) a = 2/3, b = $e^{2/3}$
(C) a = 1/3, b = $e^{1/3}$
(D) a = 1/3, b = $e^{2/3}$

8. A fair coin is tossed four times. Let X denotes the number of times a head is followed immediately by a tail. Then,

(A) mean of X is 3/8
(B) variance of X is 5/16
(C) Expectation value of X^2 is 6/7
(D) Mean of X is 7/8

9. Common roots of the equation $z^3 + 2z^2 + 2z + 1 = 0$ and $z^{149} + z^{100} + 1 = 0$ are

(A) 1
(B) $-\omega$
(C) ω^2
(D) -1

SECTION II

ASSERTION-REASON TYPE

This question contains 4 reasoning type questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct. 3 MARKS will be awarded for correct answer. 1 MARK will be deducted for wrong answer. NO MARKS WILL BE GIVEN OR DEDUCTED IF A QUESTION IS NOT ANSWERED.

10. **STATEMENT-1:** a, b, c are in H.P. and a > b > c >0 then value of $\frac{2a-b}{2a+b} + \frac{2c-b}{2c+b}$ lies between 2/3 and 1.

STATEMENT-2: 1/a, 1/b, 1/c are in A.P. and 1/a < 1/b < 1/c or b/a, 1, b/c are in A.P. or (1-d), 1, (1+d) are b/a, 1, b/c where 0 < d < 1

(a) Statement-1 is True, Statement -2 is true; Statement -2 is a correct explanation for Statement 1
(b) Statement -1 is True, Statement -2 is true; Statement -2 is NOT a correct explanation for Statement -1
(c) Statement -1 is True, Statement -2 is False
(d) Statement -1 is False, Statement -2 is true

11. **STATEMENT-1:** a, b, c are in H.P. and a > b > c >0 then value of $\frac{2a-b}{2a+b} + \frac{2c-b}{2c+b}$ lies between 2/3 and 1.

STATEMENT-2: 1/a, 1/b, 1/c are in A.P. and 1/a < 1/b < 1/c or b/a, 1, b/c are in A.P. or (1-d), 1, (1+d) are b/a, 1, b/c where 0 < d < 1

(a) Statement-1 is True, Statement -2 is true; Statement -2 is a correct explanation for Statement 1

(b) Statement -1 is True, Statement -2 is true; Statement -2 is NOT a correct explanation for Statement -1

(c) Statement -1 is True, Statement -2 is False

(d) Statement -1 is False, Statement -2 is true

12. **STATEMENT-1:** If P (A) = 3/4 and P (B) = 3/8, then P (AUB) ≥ ¾

STATEMENT-2: P(A) ≤ P(AUB) and P(B) ≤ P(AUB); hence P(AUB) ≥ max{P(A), P(B)}.

(a) Statement-1 is True, Statement -2 is true; Statement -2 is a correct explanation for Statement 1

(b) Statement -1 is True, Statement -2 is true; Statement -2 is NOT a correct explanation for Statement -1

(c) Statement -1 is True, Statement -2 is False

(d) Statement -1 is False, Statement -2 is true

13. **STATEMENT-1:** Let the direction ratios of a line L be 3, 2, -2 and L passes through the point A(6, 7, -7). Then the magnitude of the perpendicular distance of P from L is 7.

STATEMENT-2: Perpendicular distance of P from a line L is equal to the projection of PA on L.

(a) Statement-1 is True, Statement -2 is true; Statement -2 is a correct explanation for Statement 1

(b) Statement -1 is True, Statement -2 is true; Statement -2 is NOT a correct explanation for Statement -1

(c) Statement -1 is True, Statement -2 is False

(d) Statement -1 is False, Statement -2 is true

SECTION III

LINKED COMPREHENSION TYPE

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has four choices (a), (b), (c) and (d), out of which ONLY ONE is correct. 4 MARKS will be awarded for correct answer. 1 MARK will be deducted for wrong answer. NO MARKS WILL BE GIVEN OR DEDUCTED IF A QUESTION IS NOT ANSWERED.

Paragraph for Questions numbers 14 to 16

In a triangle ABC, coordinates of A are (1, 2) and the equations to medians through B and C are $x + y = 5$ and $x = 4$ respectively. Then,

14. The coordinates of the vertex C are

(A) (5, 3)
(B) (3, 4)
(C) (4, 3)
(D) (3, 5)

15. Length of the perpendicular drawn from the centroid to the side BC is

(A) $\sqrt{(17/18)}$
(B) $\sqrt{(18/17)}$
(C) $2\sqrt{(17/18)}$
(D) $2\sqrt{(18/17)}$

16. Area of the triangle ABC is

(A) 7
(B) 8
(C) 9
(D) 10

Paragraph for Questions numbers 17 to 19

Consider n G.P.'s say $GP_1, GP_2, ..., GP_n$ whose first terms are $a_1, a_2, a_3, ..., a_n$ and the common ratios are $r_1, r_2, ..., r_n$. Further, let $1>r_1>r_2> ... > r_n$. Let $S_1, S_2, ..., S_n$ stands for the sum to n terms of $GP_1, GP_2, ..., GP_n$. And $\sum_1, \sum_2, ..., \sum_n$ denote the sum to (n +1) terms of $GP_1, GP_2, ..., GP_n$.

17. S_i, $\sum_i$ are given (respectively) by

(A) $S_i = a_i\left(\frac{1-r_i^{n+1}}{1-r_i}\right)$, $\quad \sum_i = a_i\left(\frac{1-r_i^{n}}{1-r_i}\right)$
(B) $S_i = a_i\left(\frac{1-r_i^{n+1}}{1-r_i}\right)$, $\quad \sum_i = a_i\left(\frac{1-r_i^{n+2}}{1-r_i}\right)$
(C) $S_i = a_i\left(\frac{1-r_i^{n}}{1-r_i}\right)$, $\quad \sum_i = a_i\left(\frac{1-r_i^{n+1}}{1-r_i}\right)$
(D) NONE OF THESE

18. Which of the following is/are true?

(A) $\frac{\Sigma_1}{S_1} > \frac{\Sigma_2}{S_2} > \frac{\Sigma_3}{S_3} > \dots > \frac{\Sigma_n}{S_n}$

(B) $\frac{\Sigma_1}{S_1} < \frac{\Sigma_2}{S_2} < \frac{\Sigma_3}{S_3} < \dots < \frac{\Sigma_n}{S_n}$

(C) $\frac{\Sigma_1}{S_1}, \frac{\Sigma_2}{S_2}, \frac{\Sigma_3}{S_3}, \dots, \frac{\Sigma_n}{S_n}$ is neither increasing nor decreasing

(D) NONE OF THESE

19. Let all the G.P.'s be infinite G.P.'s and ak = 2k – 1 and rk = 2/(2k + 1) for k = 1,2,3, ... Then, $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{S_k S_{k+1} S_{k+2}}$ is equal to

(A) 1/120

(B) 1/60

(C) 1/30

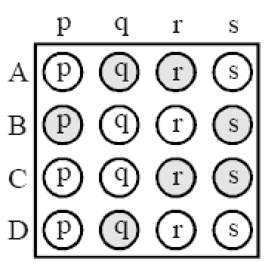
(D) NONE OF THESE

SECTION-IV

Matrix Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements in **Column I** are labelled as A, B, C and D whereas statements in **Column II** are labelled as p, q, r and s. The answers to these questions have to be appropriately bubbled asillustrated in the following example.

If the correct matches are A-q, A-r, B-p, B-s, C-r, C-s and D-q, then the correctly bubbled matrix will look like the following :



20. z1, z2, z3 represents distinct points A, B, C in Argand's plane. Match Column I with Colum II.

Column I	Column II
(a) $2z_1z_3 = z_1z_2 + z_2z_3$	(p) A, B, C are collinear
(b) $(z_3 - z_1)(z_3 - z_2) + (z_3 - z_1)(z_3 - z_2) = 0$	(q) A, B, C make equilateral triangle
(c) $\lambda_1, \lambda_2, \lambda_3 \in R$, $\lambda_1 z_1 + \lambda_2 z_2 + \lambda_3 z_3 = 0$, $\lambda_1 + \lambda_2 + \lambda_3 = 0$	(r) A, B, C lie on circle which passes through O (0,0)
(d) $\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$	(s) None of these

21. Match the following:

Column I	Column II
(a) If f (x) + 2 f (6 – x) = x for all x ϵ R. Then value of f (1) is	(p) 1
(b) Reciprocal of volume of the tetrahedron with vertices at (0,0,0), (1,1,1), (2,1,1) and (1,2,1) is	(q) 3
(c) The maximum value of f(x) where f(x) = $\int_0^x \sin\{x(1-x)\}\,dx$ occurs at x =	(r) 6
(d) If $\frac{\cos 3x}{\cos x} = \frac{1}{3}$ for some angle x, 0 < x < π/2, then $\frac{\cos 3x}{\cos x} = \frac{k}{k-4}$ for same x, where k is	(s) 7

22. Match the following:

Column I	Column II
(a) cos π/7 cos 2π/7 cos 4π/7	(p) π/3
(b) $\cos^2 3\pi/5 + \cos^2 4\pi/5$	(q) 0
(c) $S_\infty = \cos\frac{\pi}{3} + \frac{1}{2}\cos\frac{2\pi}{3} + \frac{1}{3}\cos\frac{3\pi}{3} + \ \dots\ \infty$	(r) 3/4
(d) $S_\infty = \sin\frac{\pi}{3} + \frac{1}{2}\sin\frac{2\pi}{3} + \frac{1}{3}\sin\frac{3\pi}{3} + \ \dots\ \infty$	(s) -1/8

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