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IIT JEE 2009 Test Series 3
MATHEMATICS PART-I

SECTION I

STRAIGHT OBJECTIVE TYPE

This section contains 6 multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which ONLY ONE is correct. 3 MARKS will be awarded for correct answer. 1 MARK will be deducted for wrong answer.

1. The condition that the line $L = r(a\cos\theta + b\sin\theta)$ touches the circle $r = 2c\cos\theta$ is

   (A) $b^2c^2 + 2acL - L^2 = 0$
   (B) $b^2c^2 - 2acL - L^2 = 0$
   (C) $b^2c^2 - 2acL + c^2 = 0$
   (D) $b^2c^2 + 2acL + c^2 = 0$

2. $$\lim_{x\to\pi/2} \frac{\sin x - (\sin x)^{\sin x}}{1 - \sin x + \ln \sin x} =$$

   (a) 1
   (b) -1
   (c) 2
   (d) -2

3. The number of integral solutions of **2x + 3y = 2001** is

   (a) 333
   (b) 334
   (c) 667
   (d) 665

4. From a heap containing 12 pairs of shoes if 8 shoes are randomly selected, what is the probability that there is no complete pair ?

(A) $\{{}^{24}C_8 - {}^{12}C_8 * 2^8\} / {}^{24}C_8$

(B) $\{{}^{12}C_8 / {}^{24}C_8\} * 2^8$

(C) $\{({}^{24}C_8 - {}^{8}C_4) / {}^{24}C_8\} * 2^4$

(D) $\{{}^{8}C_4 / {}^{24}C_8\} * 2^4$

5. If $f(x) = \min\{\sin x, \cos x\}$ then

$$\int_0^{\pi} f(x)\,dx =$$

(A) 0 (B) $1 - \sqrt{2}$ (C) $1 + \sqrt{2}$ (D) 1

6. The value of

$$\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$$

(A) i (B) 1 (C) – 1 (D) – i

SECTION II

MULTIPLE CHOICE TYPE QUESTIONS

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct. NO NEGATIVE marking.

7.

If x, y, z are respectively the perpendiculars from circumcentre to the sides of the triangle ABC then

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z}$$

equals

(A) 2tanAtanBtanC
(B) 2(tanA + tanB + tanC)
(C) (abc)/(4xyz)
(D) tanA + tanB + tanC

8. Let $f(x) = 2x + 2$, $0 \le x \le 4$, $2 \le f(x) \le 10$; $g(x) = 8 - 4x$, $0 \le x \le 2$, $0 \le g(x) \le 8$. If D_1, D_2 are the domain and R_1, R_2 are the range of f(x) and g(x) respectively, then

(A) $f(g(x))$ is possible ONLY when $D_1 \cap R_2 \neq \phi$
(B) $g(f(x))$ is possible ONLY when $D_2 \cap R_1 \neq \phi$
(C) $g(f(x))$ is NOT possible ONLY when $D_2 \cap R_1 \neq \phi$
(D) $g(f(x))$ is NOT possible ONLY when $D_2 \cap R_1 \neq \phi$

9. Let

$$\left| \frac{z-1}{z+1} \right| = \text{amp}\left(\frac{z-1}{z+1}\right) = k$$

where $z = x + iy$ and $k \neq 1, -1$. If they represent two circles of the form $x^2 + y^2 + 2\alpha x + 1 = 0$ and $x^2 + y^2 + 2\beta x - 1 = 0$ then,

(A) The circles are orthogonal.
(B) $\alpha(k^2-1)-(k^2+1) = 0$.
(C) $1 + \beta \tan(k) = 0$.
(D) Represents a point circle if

$$\frac{k^2+1}{k^2-1} - \frac{1}{\tan k} = 0$$

10.

If

$$B = \begin{pmatrix} 6 & 9 \\ -4 & -6 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$$

then.

(A) BC=CB
(B) B doesn't have an INVERSE
(C) B and C both has an INVERSE
(D) INVERSE of C exists.

SECTION III

ASSERTION-REASON TYPE

This question contains 4 reasoning type questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct. 3 MARKS will be awarded for correct answer. 1 MARK will be deducted for wrong answer. NO MARKS WILL BE GIVEN OR DEDUCTED IF A QUESTION IS NOT ANSWERED.

11.

Consider the functions $f(x)$ and $g(x)$ defined as

$$f(x) = \cos(\sin x) \text{ and } g(x) = \sin(\cos x)$$

STATEMENT-1: $f(x)$ is greater than zero for all real x.

STATEMENT-2: $f(x) - g(x)$ is minimum at $x = \pi$.

(a) STATEMENT-1 is True, STATEMENT-2 is true; STATEMENT-2 is a correct explanation for STATEMENT 1
(b) STATEMENT-1 is True, STATEMENT-2 is true; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(c) STATEMENT-1 is True, STATEMENT-2 is False
(d) STATEMENT-1 is False, STATEMENT-2 is true

12.

Let $f(x)=|x-1|\{[x]-[-x]\}$, for $x\neq 1$; $f(x)=0$ for $x=1$.

STATEMENT-1: f(x) is differentiable at x = 1

STATEMENT-2: $f'(1^+) = f'(1^-)$

(A) STATEMENT-1 is True, STATEMENT-2 IS True; STATEMENT -2 is a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 IS True; STATEMENT -2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 IS False
(D) STATEMENT-1 is False, STATEMENT-2 IS True

13.

STATEMENT-1: Let $a \neq b$, $c \neq 0$. If the quadratic equations, $x^2+ax+bc=0$ and $x^2+bx+ca=0$, have a common root, then the other roots satisfy $x^2+cx+ab=0$.

STATEMENT-2: b is the common root.

(e) STATEMENT-1 is True, STATEMENT-2 is true; STATEMENT-2 is a correct explanation for STATEMENT 1

(f) STATEMENT-1 is True, STATEMENT-2 is true; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

(g) STATEMENT-1 is True, STATEMENT-2 is False

(h) STATEMENT-1 is False, STATEMENT-2 is true

14.

STATEMENT-1: a, b, c are in H.P. and $a > b > c > 0$then value of

$$\frac{2a-b}{2a+b}+\frac{2c-b}{2c+b}$$

lies between 2/3 and 1.

STATEMENT-2: 1/a,1/b, 1/c are in A.P. and $1/a < 1/b < 1/c$ or b/a, 1, b/c are in A.P. or (1-d), 1, (1+d) are b/a, 1, b/c where $0 < d < 1$.

STATEMENT-1 is True, STATEMENT-2 is true; STATEMENT-2 is a correct explanation for STATEMENT 1

(a) STATEMENT-1 is True, STATEMENT-2 is true; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

(b) STATEMENT-1 is True, STATEMENT-2 is False

(c) STATEMENT-1 is False, STATEMENT-2 is true

SECTION IV

LINKED COMPREHENSION TYPE

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has four choices (a), (b), (c) and (d), out of which ONLY ONE is correct. 4 MARKS will be awarded for correct answer. 1 MARK will be deducted for wrong answer. NO MARKS WILL BE GIVEN OR DEDUCTED IF A QUESTION IS NOT ANSWERED.

Paragraph for Questions numbers 15 to 17

Let p be an interior angle of a triangle ABC and let BP and CP meet AC and AB in E and F respectively. Denote by [X] the area of figure X, whether triangle or quadrilateral, as the case may be. Area of the triangle BPF is 4 units, that of triangle BPC is 8 units.. also area of triangle CPE = 133 units.

Q15. What is the value in square units of [APF]?

A. 52
B. 91
C. 39
D. 13

Q16. What is the value in square units of [AEPF]?

A. 52 B. 39 C. 137 D. 143

Q17. How many ordered triplet solutions of (x, y, z) does the system $x + y + z = 4$, $x^2 + y^2 + z^2 = 14$, $x^3 + y^3 + z^3 = 34$ have?

A. Six B. One C. Eighteen D. Thirty six

Paragraph for Questions numbers 18 to 20

Let *AD, BE* and *CF* be the angular bisectors of the interior angles of triangle *ABC*. These bisectors are concurrent at a point *I* called incentre of the triangle. We know from geometry that *BD. AC = AB. DC.*

If $BC = \alpha$, $CA = \beta$, $AB = \gamma$ with reference to some origin. Let ***a, b, c*** be the positions vectors of A, B, C respectively. Then

Q18. The value of $\alpha\boldsymbol{IA} + \beta\boldsymbol{IB} + \gamma\boldsymbol{IC}$ must be

(A) ZERO (B) ONE (C) $\alpha+\beta+\gamma$ (D)NONE OF THESE

Q19. If r is perpendicular distance of I from the side *BC* then ***IB.IC*** must be

(A) $r^2 \operatorname{cosec} B/2 \operatorname{cosec} C/2$
(B) $r^2 \operatorname{cosec} B/2 \operatorname{cosec} C/2 \sin A/2$
(C) $-r^2 \operatorname{cosec} B/2 \operatorname{cosec} C/2 \sin A/2$
(D) NONE OF THESE

Q20. If bisectors of exterior angles B and C meet at a point I_1, outside the triangle ABC then position vector of I_1 must be

(A) $(\alpha \boldsymbol{a} - \beta \boldsymbol{b} - \gamma \boldsymbol{c})/(\alpha - \beta - \gamma)$
(B) $(- \alpha \boldsymbol{a} + \beta \boldsymbol{b} - \gamma \boldsymbol{c})/(- \alpha + \beta + \gamma)$
(C) $(\boldsymbol{b} + \boldsymbol{c})/(\alpha + \beta + \gamma)$
(D) NONE OF THESE

Paragraph for Questions numbers 21 to 23

Let $f(x) = (x-a)(x-b)-((a+b)/2)$ and $f(x) = 0$ has both non negative roots.

Q21. Which of the following is true?

(A) $(a+b) > 0$ and $ab - (a+b)/2 \geq 0$
(B) $(a+b) < 0$ and $ab - (a+b)/2 \geq 0$
(C) $(a+b) > 0$ and $ab - (a+b)/2 \leq 0$
(D) $(a+b) < 0$ and $ab - (a+b)/2 \leq 0$

Q22. When $f'(x) = 0$ then,

(A) $f(x)$ will have a maximum at $x = (a + b)/2$
(B) $f(x)$ will have a maximum at $x = (a - b)/2$
(C) $f(x)$ will have a minimum at $x = (a + b)/2$
(D) $f(x)$ will have a minimum at $x = (a - b)/2$

Q23. $f(x) + (a + b)^2/4$ will be

(A) Less than or equal to ZERO.
(B) Less than ZERO.
(C) Greater than ZERO.
(D) Greater than or equal to ZERO