Physics

Paragraph for comprehension 1 to 3

A ball of radius R carries a positive charge whose volume charge density depends only on the distance r from the ball's centre as $\rho = \rho_0\left(1 - \frac{r}{R}\right)$ where ρ_0 is a constant. Answer the following questions:

1. The magnitude of electric field as a function of the distance r inside the ball is given by

   (a) $E = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$

   (b) $E = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{4} - \frac{r^2}{3R}\right)$

   (c) $E = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3} + \frac{r^2}{4R}\right)$

   (d) $E = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{4} + \frac{r^2}{3R}\right)$

2. The magnitude of electric field as a function of distance r outside the ball is given by

   (a) $E = \frac{\rho_0 R^3}{8\varepsilon_0 r^2}$

   (b) $E = \frac{\rho_0 R^3}{12\varepsilon_0 r^2}$

   (c) $E = \frac{\rho_0 R^3}{16\varepsilon_0 r^2}$

   (d) $E = \frac{\rho_0 R^3}{24\varepsilon_0 r^2}$

3. The maximum electric field intensity is

   (a) $E_{\max} = \frac{\rho_0 R}{9\varepsilon_0}$

   (b) $E_{\max} = \frac{\rho_0 R}{6\varepsilon_0}$

   (c) $E_{\max} = \frac{\rho_0 R}{3\varepsilon_0}$

   (d) $E_{\max} = \frac{\rho_0 R}{\varepsilon_0}$

Chemistry

Paragraph for comprehension 4 to 6

An electrical source which might be used on a spacecraft should be light weight. Yet deliver high voltage and maximum useful energy. A galvanic cell using aluminum, one of the lightest metal, might be considered.

$Al \mid Al^{3+} \mid\mid Ce^{4+} \mid Ce^{3+} \mid Pt$

The cell is constructed with 2.0L 4M $Al_2(SO_4)_3$ and 2.0L 6M $Ce(SO_4)_2$. If $E_0(Al^{3+}/Al) = -1.76$ V and $E_0(Ce^{4+}/Ce^{3+}) = 1.443$ V, answer the following questions:

1. Determine emf of the cell in the given conditions, if cathode compartment is initially 1.0 M in Ce^{3+} ion.
   (a) 3.203 V
   (b) 3.23 V
   (c) 3.177 V
   (d) zero
2. What is the maximum amount of useful electrical energy which could be derived from this cell?
   (a) –3708 V
   (b) – 4308 V
   (c) – 5308 V
   (d) – 6308 V
3. The instruments on the aircraft require a reasonably steady voltage to operate correctly. What will be the cell voltage when the concentration of Ce^{4+} ion has dropped to half of its original concentration? Assume this galvanic cell operates under a reversible condition at 25 ^{0}C.
   a) 3.203 V
   b) 3.23 V
   c) 3.177 V
   d) zero

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Mathematics

Paragraph for comprehension 7 to 9

AB and CD are major and minor axes of an ellipse respectively which circumscribes a rectangle PQRS having side length as 8 and 6 units. The centre of ellipse being origin. Answer the following questions.

7. Which of the following cannot be the vertex of given ellipse?
   (a) (5, 0)
   (b) (6, 0)
   (c) (8, 0)
   (d) (10, 0)
8. Co- normal points are points on an ellipse normals drawn at which are concurrent. The number of set of four co- normal points out of A, B, C, D and P, Q, R, S is
   (a) 1
   (b) ${}^{8}C_{4}$
   (c) 5
   (d) 6
9. If eccentricity of ellipse is 1/2 , then area of the ellipse is
   (a) $14\sqrt{3}\pi$
   (b) $7\sqrt{3}\pi$
   (c) 21π
   (d) none of these

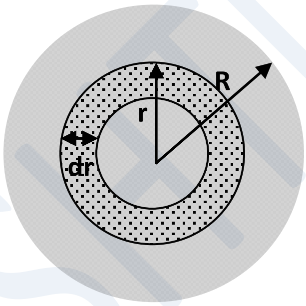
SOLUTIONS:

1. **(a)**
2. **(b)**
3. **(a)**
4. **(b)**
5. **(a)**
6. **(c)**
7. **(a)**
8. **(c)**
9. **(a)**

EXPLANATIONS

Physics

1. The given charge distribution in the ball is not uniform but varies wrt distance from the centre. In order to calculate the electric field due to it, the ball can be assumed to be made of various concentric shells. Let us consider one such spherical shell having radius r and thickness dr.
   Volume of this elementary spherical shell = $4\pi r^2 dr$
   (How?)
   Hence, charge contained in this volume



$$dq = (volume) \times (charge\ density)$$

$$\Rightarrow dq = (4\pi r^2 dr)\rho = (4\pi r^2 dr)\left[\rho_0\left(1-\frac{r}{R}\right)\right]$$

$$\Rightarrow dq = 4\pi\rho_0\left[\left(1-\frac{r}{R}\right)\right]r^2 dr \qquad ...(1)$$

Hence, charge contained within the volume of a sphere of radius r (r < R) is

$$q = \int_0^r 4\pi\rho_0\left[\left(1-\frac{r}{R}\right)\right]r^2 dr$$

$$\Rightarrow q = 4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^4}{4R}\right] \qquad ...(2)$$

Now the electric field E at a distance r (r < R) from the centre of ball can be calculated as if the charge q is concentrated at the centre of the ball.

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^4}{4R}\right]}{r^2}$$

$$\Rightarrow E = \frac{\rho_0}{\varepsilon_0}\left[\frac{r}{3}-\frac{r^2}{4R}\right] \qquad ...(A)$$

2. For calculating the electric field outside the ball, we should calculate first the total charge present in the ball. Integrating equation (1) of previous part for total charge in the ball, we have

$$Q = \int_0^R 4\pi\rho_0 \left[\left(1 - \frac{r}{R}\right)\right] r^2 dr$$

$$\Rightarrow Q = 4\pi\rho_0 \left[\frac{R^3}{3} - \frac{R^4}{4R}\right]$$

$$\Rightarrow Q = \frac{\pi\rho_0 R^3}{3} \qquad ...(3)$$

Again to find the electric field outside the ball, total charge Q will be considered to be concentrated at the centre. Hence, electric field at a distance r (r > R) from the centre of the ball is

$$\therefore \text{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{\pi\rho_0 R^3}{3}}{r^2}$$

$$\Rightarrow E = \frac{\rho_0 R^3}{12\varepsilon_0 r^2} \qquad ...(B)$$

3. It is clear from expressions of E from equation (B) that E i decreases for r > R. Therefore E must be maximum inside the sphere or on the surface of the sphere. Let us see.

At maximum intensity

$$\frac{dE}{dr} = 0$$

$$\Rightarrow \frac{d\left(\frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)\right)}{dr} = 0 \qquad (from\ (A))$$

$$\Rightarrow \frac{\rho_0}{\varepsilon_0}\left(\frac{1}{3} - \frac{r}{2R}\right) = 0$$

$$\Rightarrow r = \frac{2R}{3}$$

Hence,

$$E_{\max} = \frac{\rho_0}{\varepsilon_0}\left[\frac{r}{3} - \frac{r^2}{4R}\right] = \frac{\rho_0}{\varepsilon_0}\left[\frac{2R/3}{3} - \frac{(2R/3)^2}{4R}\right]$$

$$\Rightarrow E_{\max} = \frac{\rho_0 R}{9\varepsilon_0}$$

Chemistry

4. The cell reaction is

$$Al + 3Ce^{4+} \rightarrow Al^{3+} + 3Ce^{3+}$$

$$E^0{}_{cell} = E^0{}_{cathode} - E^0{}_{anode} = +1.443 + 1.76 = 3.203V$$

$$E^0{}_{cell} = E^0{}_{cell} - \frac{0.059}{3}\log\frac{[Ce^{3+}]^3[Al^{3+}]}{[Ce^{4+}]^3} = 3.203 - \frac{0.059}{3}\log\frac{8}{6^3} = 3.23V$$

5. Since, Ce^{4+} is in limited quantity, therefore,

$$\Delta G^0 = -nE^0F = -(1\ mol\ e\ per\ mol\ Ce^{4+})(3.203)(96500) = -309kJ/mol\ Ce^{4+}$$

$$\Rightarrow \Delta G^0{}_{total} = -309 \times 12 = -3708K$$

6. Initially $[Al^{3+}]$ = 8 M, $[Ce^{4+}]$ = 6 M, $[Ce^{3+}]$ = 1 M

Also total mole of Ce^{4+} present initially = 6 x 2 = 12

Total mole of Al^{3+} present initially = 8 x 2 = 16

Total mole of Ce^{3+} present initially = 2

As the concentration of Ce^{4+} drops to half, 6 mol of Ce^{4+} will be converted to Ce^{3+}.

⇨ $[Ce^{4+}]$ = 3 M, $[Ce^{3+}]$ = 8/2 = 4 M

Also, for three mol Ce^{4+} reduction 1 mol of Al^{3+} comes in solution.

⇨ For 6 mol of Ce^{4+} reduction, two mol of Al^{3+} comes in solution

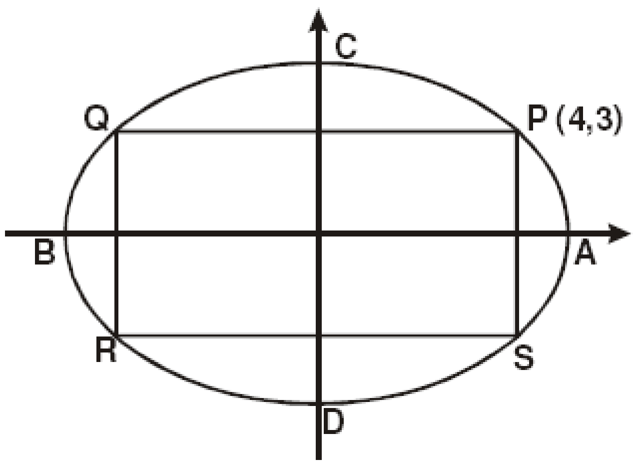
⇨ $[Al^{3+}]_{final}$ = 9 M

$$\therefore E^0{}_{cell} = E^0{}_{cell} - \frac{0.059}{3}\log\frac{[Ce^{3+}]^3[Al^{3+}]}{[Ce^{4+}]^3}$$

$$\Rightarrow E^0{}_{cell} = 3.203 - \frac{0.059}{3}\log\frac{[4]^3[9]}{3^3} = 3.177V$$

Mathematics

Let a possible figure be as



let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Point (4, 3) lie on the ellipse

$\therefore \quad \frac{16}{a^2}+\frac{9}{b^2}=1 \qquad ...(1)$

7.

Infinitely many ellipses are possible and a can take any value more than 4. But when a = 5, we get b = 5 which gives us a circle. For vertices of an ellipse to exist, it cannot be a circle.

8.

Clearly, due to symmetry, normals at any two points out of P, Q, R, S, will meet on the principal axes of the ellipse. Hence, the possible set of 4 co-normal points are PQCD, RSCD, PSAB, QRAB and ABCD.

9.

If $e=\frac{1}{2}$ then by $e^2=1-\frac{b^2}{a^2}$, we get

$\frac{b^2}{a^2}=1-\frac{1}{4}=\frac{3}{4} \quad \Rightarrow \quad 4b^2=3a^2$ ....(2)

Solving (1) & (2), we get

$a=2\sqrt{7},\ b=\sqrt{21}$

Area of ellipse

$=\pi ab=\pi\left(2\sqrt{2}\right)\sqrt{21}=14\sqrt{3}\pi$