

DAILY TEST SERIES FOR IIT-JEE 2009 FROM VIDYA DRISHTI

20.03.2009

Total time: 20 min

Multiple choice questions (one or more options correct)

**Physics**

A ball of mass  $m$  having charge  $q$  is projected from the ground with an initial velocity  $u$  and angle of projection  $\theta$ . At the time of projection a vertically upward electric field  $E$  is switched on. When it reaches the maximum height the electric field is switched off and a horizontal electric field of same magnitude  $E$  is switched on in the direction of velocity. Then, answer the following questions:

- If the time of flight to the maximum height is  $t_1$  and the time of flight from maximum height to ground is  $t_2$  then  
 (a)  $t_1 = t_2$       (b)  $t_1 < t_2$       (c)  $t_1 > t_2$       (d) more information is required to answer this

- The maximum height  $h_{\max}$  reached by the projectile is

- $\frac{u^2 \sin^2 \theta}{2g}$
- $\frac{u^2 \sin^2 \theta}{2\left(g - \frac{qE}{m}\right)}$
- $\frac{u^2 \sin^2 \theta}{2\left(g + \frac{qE}{m}\right)}$
- $\frac{u^2 \sin^2 \theta}{2g} - \frac{mu^2 \sin^2 \theta}{2qE}$

- The horizontal velocity of the ball when it again strikes the ground is

- $u \cos \theta$
- $u \cos \theta - \frac{qE}{m} \sqrt{\frac{2h_{\max}}{g}}$
- $u \cos \theta + \frac{qE}{m} \sqrt{\frac{2h_{\max}}{g}}$
- none of these

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**Chemistry**

Paragraph

One mole of an ideal gas originally at a volume of 8.0 L at 1000 K, is allowed to expand adiabatically until final volume is 16.00 L. For the gas  $C_v = 1.5 R$ . Answer the following questions

4.  $\Delta S$  for the process when the expansion takes place reversibly is
- (a) 0                      (b) 3.06 J/K                      (c) 5.76 J/K                      (d) 7.36 J/K
5.  $\Delta S$  for the process when the expansion takes place against a constant pressure of 3.00 atm is
- (a) 0                      (b) 3.06 J/K                      (c) 5.76 J/K                      (d) 7.36 J/K
6.  $\Delta S$  for the process when the change in volume involves a free expansion is
- (a) 0                      (b) 3.06 J/K                      (c) 5.76 J/K                      (d) 7.36 J/K

**Mathematics**

Consider a rectangular hyperbola  $xy = c^2$ . The normals at  $(x_i, y_i)$ ,  $i = 1, 2, 3, 4$  meet the point  $(\alpha, \beta)$ . Then

7. Value of  $\sum x_i$  is
- (a)  $\alpha$   
(b)  $\beta$   
(c)  $c$   
(d) none of the above
8. Value of  $\sum y_i$  is
- (e)  $\alpha$   
(f)  $\beta$   
(g)  $c$   
(h) none of the above
9. Value of  $\sum x_i^2 + \sum y_i^2$  is
- (i)  $\alpha^2$   
(j)  $\beta^2$   
(k)  $c^2$   
(l)  $\alpha^2 + \beta^2$

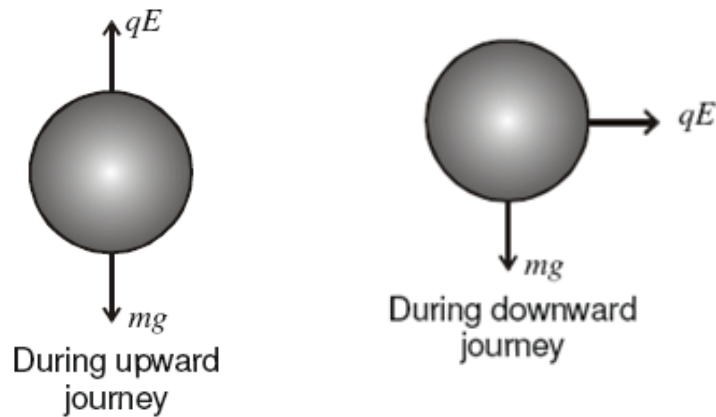
20.03.2009

**SOLUTION:**

**Physics**

**1. (c)**

As vertical motion is independent of horizontal motion, we have  $t_1 > t_2$ .  
Retardation during upward journey is less than the acceleration during downward journey.  
FBD is shown below.



**2. (b)**

We know that when a projectile is projected with an initial velocity  $u$  and angle  $\theta$  with the horizontal, maximum height is given by:

$$H_{\max} = \frac{u^2 \sin^2 \theta}{2g} \quad \dots(1)$$

Here, we can't use  $g$  in denominator. But we can still use this equation but with different term (acceleration) instead of  $g$ .

Let us find net acceleration in downward direction during upward motion.

During upward motion,

$$mg - qE = ma$$

$$\Rightarrow a = g - \frac{qE}{m}$$

We can put this equation in main equation of maximum height of projectile

Putting this value of  $a$  in place of  $g$  in equation (1), we get maximum height as

$$H_{\max} = \frac{u^2 \sin^2 \theta}{2 \left( g - \frac{qE}{m} \right)} \quad \dots(2)$$

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20.03.2009

3. (c)

Now, we use equation of motion in horizontal direction.

Clearly, horizontal velocity of ball up to maximum height is  $u \cos \theta$ , as there is no acceleration (or force) in the horizontal direction. Electric field in the horizontal direction is switched after maximum height. Therefore, there will be acceleration in the right direction. Clearly,

$$a_x = \frac{qE}{m}$$

Therefore,  $v_x = u_x + a_x t_2$  where  $t_2$  is the time taken by the ball during downward motion.

Clearly, time taken by the ball to reach the ground from the maximum height to the ground is

$$t_2 = \sqrt{\frac{2H_{\max}}{g}}$$

Here we have again used the concept that vertical motion is independent of horizontal motion. Therefore, we have find out the time in vertical direction during downward motion of the ball.

Hence,

$$v_x = u_x + a_x t_2$$

$$\Rightarrow v_x = u \cos \theta + \frac{qE}{m} \sqrt{\frac{2H_{\max}}{g}}$$

Chemistry

4. (a)

In case of reversible adiabatic process,

$$\Delta Q_{\text{rev}} = 0$$

$$\Rightarrow \Delta S = 0$$

5. (b)

In case of irreversible adiabatic expansion:

$$\Delta Q = 0$$

$$\Rightarrow P_{\text{ext}} V_2 - V_1 + n C_v T_2 - T_1 = 0$$

$$\Rightarrow T_2 = \frac{P_{\text{ext}}(V_1 - V_2)}{nC_v} + T_1 = \frac{-3 \times 8}{0.0821 \times 1.5} + 1000 = 805 \text{ K}$$

$$\therefore \Delta S = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} = 1.5 \times 8.314 \ln \frac{805}{1000} + 8.314 \ln 2$$

$$\Rightarrow \Delta S = 3.06 \text{ JK}^{-1}$$

20.03.2009

**6. (c)**

In case of free adiabatic expansion  $P_{\text{ext}} = 0$

$$\Delta Q = 0$$

$$\Rightarrow P_{\text{ext}} \Delta V + nC_v T_2 - T_1 = 0$$

$$\Rightarrow T_1 = T_2 = 1000K$$

$$\therefore \Delta S = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} = 1.5 \times 8.314 \ln \frac{1000}{1000} + 8.314 \ln 2$$

$$\Rightarrow \Delta S = 5.76 JK^{-1}$$

**Mathematics**

Let  $x_i, y_i = \left( ct_i, \frac{c}{t_i} \right)$ ,  $i = 1, 2, 3, 4$  are the points on the rectangular hyperbola  $xy = c^2$ .

Equation of normal to the hyperbola  $xy = c^2$  at  $\left( ct_i, \frac{c}{t_i} \right)$  is

$$ct^4 - t^3x + ty - c = 0.$$

Since, it passes through  $(\alpha, \beta)$ , therefore,

$$ct^4 - t^3\alpha + t\beta - c = 0$$

Clearly this equation is a biquadratic in  $t$ . Let the roots of the equation are  $t_1, t_2, t_3$  and  $t_4$ .

Then,

$$\sum t_i = \frac{\alpha}{c} \quad \dots(1)$$

$$\sum t_1 t_2 = 0 \quad \dots(2)$$

$$\sum t_1 t_2 t_3 = -\frac{\beta}{c} \quad \dots(3)$$

$$\sum t_1 t_2 t_3 t_4 = -1 \quad \dots(4)$$

**7. (a)**

$$\sum x_i = c \sum t_i = \alpha$$

**8. (b)**

$$\sum y_i = c \left( \sum \frac{1}{t_i} \right) = c \left( \frac{\sum t_1 t_2 t_3}{t_1 t_2 t_3 t_4} \right) = \beta$$

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20.03.2009

9. (d)

$$\sum x_i^2 = c^2 \sum t_i^2 = c^2 \sum t_i^2 - 2 \sum t_1 t_2 = \alpha^2$$

$$\sum y_i^2 = \sum y_i^2 - 2 \sum y_1 y_2$$

$$\Rightarrow \sum y_i^2 = \beta^2 - 2c^2 \sum \left( \frac{1}{t_1 t_2} \right)$$

$$\Rightarrow \sum y_i^2 = \beta^2 - 2c^2 \frac{\sum t_1 t_2}{t_1 t_2 t_3 t_4}$$

$$\Rightarrow \sum y_i^2 = \beta^2$$

$$\therefore \sum x_i^2 + \sum y_i^2 = \alpha^2 + \beta^2$$