DAILY TEST SERIES FOR IIT-JEE 2009 FROM VIDYA DRISHTI

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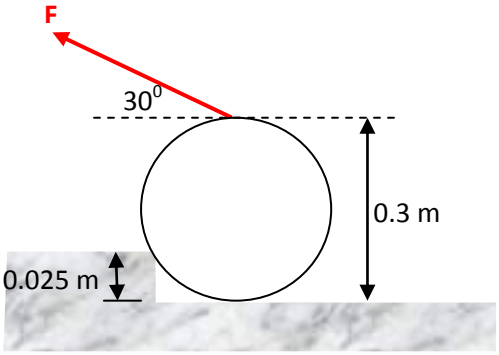
Total time: 20 min

Physics

Straight objective type questions

1. A cylinder of diameter 0.3 m and mass 25 kg rests on a rough surface as shown in figure. The coefficients of static and kinetic frictions are 0.4 and 0.35 respectively. The minimum value of force F to be applied to roll the cylinder without slip over the step is (using g = 10 m/s^2)
   (a) zero
   (b) 25.7 N
   (c) 36. 9 N
   (d) 71.4 N



2. A steel rod of length 1 m rests on a smooth horizontal base. Coefficient of linear expansion of steel = 1.2 X 10^{-5} / ^{0}C. If it is heated from 20 ^{0}C to 50 ^{0}C, what is the longitudinal strain developed?
   (a) Zero
   (b) 3.2 X 10^{-4}
   (c) 6.4 X 10^{-4}
   (d) 9.6 X 10^{-4}

Mathematics

1. {x}, [x] and (x) are the fractional part of x, greatest integer of x and nearest integer to x respectively and defined as follows:
   x = [x] + {x};

$$(x)=\begin{cases}2[x]-\{x\}, & x<0\\ [x]+3\{x\}, & x\geq 0\end{cases}$$

   Then, solutions of equation (x) = x + {x} are
   (a) All integers
   (b) All rational numbers
   (c) All whole numbers
   (d) All natural numbers

2. Sides of a triangle are in A. P. If a < min. {b, c} and c > max. {a, b}, then cos A is equal to

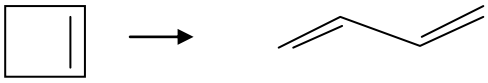
(a) $\frac{3c-4b}{2b}$ (b) $\frac{3c-4b}{2c}$ (c) $\frac{4c-3b}{2b}$ (d) $\frac{4c-3b}{2c}$

Chemistry

5. What will be the temperature difference needed in hot air balloon to lift 1.0 kg weight. Assume that the volume of balloon is 100 m^3, the temperature of atmosphere is 25^0C and pressure is 1.0 atm. Average molar mass of air is 29 amu.

(a) 3^0C (b) 5^0C (c) 15^0C (d) 25^0C

6. Determine resonance energy of 1,3-butadiene using the following information:
Enthalpy of combustion: 1, 3-butadiene = -2841 kJ/mol, C (gr) = -394 kJ/mol, H_2 (g) = -285 kJ/mol
Bond enthalpy: $\tilde{C}$ C = 348 kJ/mol, C=C = 615 kJ/mol.
Standard enthalpy of formation of cyclobutene = 130 kJ/mol.



(a) 86 kJ/mol (b) 336 kJ/mol (c) – 86 kJ/mol (d) – 336 kJ/mol

SOLUTION:

Physics

1. (d)

This question seems difficult. This question has so many useful concepts. Please go through carefully.

Basic idea here is that we have to find out the minimum value of force F to be applied to roll the cylinder without slip over the step.

If you think carefully, you will find that for this to happen cylinder must be on the verge of lifting the horizontal surface at bottom D. Hence, normal reaction at bottom will be zero.

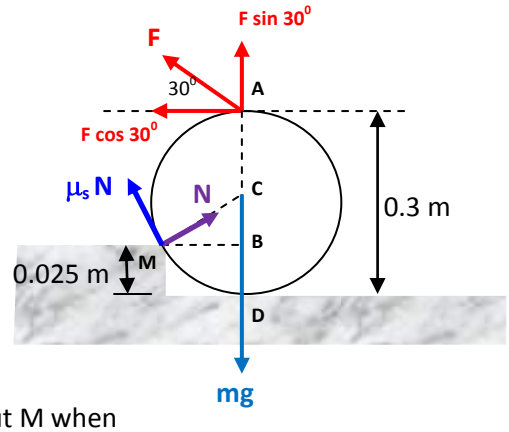
Since there is no slipping, limiting static friction will act point M for this situation.

Considering these points in our mind, let us draw the FBD of cylinder. Forces are:

Weight mg (downward)

Normal reaction N at point M along centre of the cylinder (Why?☺)

Limiting static friction μ_s N perpendicular to N as shown (why?)



Clearly, cylinder rotates about M when

(anticlockwise torque) > (clockwise torque)

For minimum value of force F, anticlockwise torque will be just greater than clockwise torque. Thus for limiting case,

(anticlockwise torque) = (clockwise torque)

⇨ $(F\cos 30^0)(AB) + (F\sin 30^0)(MB) = (mg)(MB)$ …(1)

Now from figure,

AB = AD – BD = 0.275 m

MB = $\sqrt{(MC)^2 - (CB)^2}$ = 0.08 m.

Putting these values in (1), we get, F = 71.4 N

Hence, the minimum value of force F to be applied to roll the cylinder without slip over the step is 71.4 N.

2. (a)

Why can't we use formula, strain = $\alpha\Delta T$ here?

Here, rod lies on a frictionless surface, so it can freely expand. Thus, there is no opposition to expansion. A longitudinal strain develops if and only if, there is an opposition to the expansion. Since there is no opposition in this case, hence the longitudinal stain here = Zero.

Mathematics

3. (c)

Let us consider two cases:

Case I: If $x < 0$

In this case $(x) = 2[x] - \{x\}$

Substituting the value (x), in the given equation then

$2[x] - \{x\} = x + \{x\}$

$\Rightarrow$ $2[x] - \{x\} = [x] + \{x\} + \{x\}$ (because, $x = [x] + \{x\}$)

$\Rightarrow$ $3\{x\} = [x]$ …(1)

which is not true when $x < 0$, $[x] < 0$

Therefore, $[x] = -2, -1$

For $x \in R$, $0 \le \{x\} < 1$

Therefore, $0 \le 3\{x\} < 3$

then RHS of (1) is negative real but LHS of (1) is non negative a less than 3.

A negative real number cannot be equal to a non negative real number.

Hence, given equation has no solution when $x < 0$.

Case II: If $x \ge 0$

Then $(x) = [x] + 3\{x\}$

Substituting the value (x), in the given equation then

$[x] + 3\{x\} = x + \{x\}$

$\Leftrightarrow$ $[x] + 3\{x\} = [x] + \{x\} + \{x\}$ (because, $x = [x] + \{x\}$)

⇔ {x} = 0
Therefore, x = [x] + 0
for x ≥ 0
So, x = 0, 1, 2, 3…
which is the required solution of the given equation.
Hence, solution of given equation is all whole numbers

4. (d)

Clearly, a is the smallest side. Therefore,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{4c - 3b}{2c}$$

Chemistry

5. (a)

$$\Delta m = m_{cold} - m_{hot} = [n_{cold} - n_{hot}]\, M = \frac{PVM}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

Solving we get: $\Delta T = T_1 - T_2 = 3\ ^0C$

6. (c)

From combustion data:

ΔH^o_f (1, 3-butadiene) = – 4 x 394 – 4 x 285 + 2841

⇨ ΔH^o_f (1, 3-butadiene) = 125 kJ/mol …(1)

From bond enthalpies:

2 x 348 – 615 = ΔH^o_f (1, 3-butadiene) – 130

⇨ ΔH^o_f (1, 3-butadiene) = 211 kJ/mol ...(2)

R. E. = 125 – 211 = –86 kJ/mol