DAILY TEST SERIES FOR IIT-JEE 2009 FROM VIDYA DRISHTI

14.03.2009

Total time for three comprehensions: 20 min
Linked comprehension type questions

Physics

Paragraph for comprehension 1 to 3

The electric potential V in volt in a region of space is given by $V = ax^2 + ay^2 + 2az^2$ where a is a constant having proper dimension.

1. The work done by the field when a 2 μC charge moves from a point (0, 0, 0.1) to origin is 5×10^{-5} J. Then value of a is
   (a) 10^3 V/m^2
   (b) 1.25×10^3 V/m^2
   (c) 2.25×10^3 V/m^2
   (d) 3.25×10^3 V/m^2

2. In every plane parallel to X-Y plane the equipotential lines are
   (a) Straight lines
   (b) Circles
   (c) Parabola
   (d) Hyperbola

3. Corresponding to V = 6250 V and z = √2 m we have
   (a) Focus of parabola at (0, 0, √2)
   (b) Focus of hyperbola at (0, 0, √2)
   (c) Radius of the circle of the equipotential line is 1 m
   (d) None of the above

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Chemistry

Paragraph for comprehension 4 to 6

In a laboratory class, a student is given three flasks that are labeled Q, R and S. Each flask contains one of the following solutions: 1.0 M $Pb(NO_3)_2$, 1.0 M NaCl and 1.0 M K_2CO_3. The student is also given two flasks that are labeled X and Y. One of these flasks contains 1.0 M $AgNO_3$ and other contains 1.0M $BaCl_2$.

1. When the student combined the sample of solution Q with sample of solution X, a precipitate formed. A precipitate also formed when samples of solutions Q and Y were combined. Identify Q

(a) K_2CO_3
(b) $Pb(NO_3)_2$
(c) NaCl
(d) None of the above

2. When solution Q is mixed with solution R, a precipitate forms. However, no precipitate forms when solution Q is mixed with solution S. Identify R and S.

(a) R is NaCl and S is $Pb(NO_3)_2$
(b) R is $Pb(NO_3)_2$ and S is NaCl
(c) R is K_2CO_3 and S is $Pb(NO_3)_2$
(d) R is NaCl and S is K_2CO_3

3. Identify X and Y.

(a) X is $AgNO_3$and Y is $BaCl_2$
(b) X is $BaCl_2$ and Y is $AgNO_3$
(c) X is $BaCl_2$ and Y is NaCl
(d) None of the above

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Mathematics

Paragraph for comprehension 7 to 9

The tangents at P and P' on the parabola $y^2 = 4ax$ meet in T. S is the focus, O is the vertex and SP, ST and SP' are equal to α, β and γ respectively. The tangent at the point $P(x_1, y_1)$ to the parabola $y^2 = 4ax$ meets the parabola $y^2 = 4a(x + b)$ at Q and R.

7. The normal chord at a point P on the parabola $y^2 = 4ax$ subtends a right angle at vertex O. Then the length of the chord is
   (a) $\sqrt{3}a$
   (b) $2\sqrt{3}a$
   (c) $4\sqrt{3}a$
   (d) $6\sqrt{3}a$

8. The roots of the equation $\alpha x^2 + 2\beta x + \gamma = 0$ are
   (a) Real and different
   (b) Real and equal
   (c) Complex numbers
   (d) Irrational

9. If the chord of contact of tangents from point P to the parabola $y^2 = 4ax$ touches the parabola $x^2 = 4by$, the locus of P is
   (a) circle
   (b) parabola
   (c) ellipse
   (d) hyperbola

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SOLUTION:

Physics

1. **(b)**
2. **(b)**
3. **(c)**

1. As electric field is a conservative field, therefore,
   Work done by charge = negative of change in potential energy
   $W = -\Delta U$
   ⇨ $W = -q\,\Delta V$ (ΔV = change in electric potential; $\Delta U = \Delta V\, q$)
   ⇨ **$W = -q\,(V_2 - V_1)$** … (1)
   Since, $V = ax^2 + ay^2 + 2az^2$ (given)
   Therefore, initial potential at point (0, 0, 0.1) is
   $V_1 = a\,(0)^2 + a(0)^2 + 2a(0.1)^2$
   ⇨ $V_1 = 0.02\,a$ … (2)
   Similarly, final potential at origin (0, 0, 0) is
   $V_2 = a\,(0)^2 + a(0)^2 + 2a(0)^2$
   ⇨ $V_2 = 0$ … (3)
   Also $q = 2 \times 10^{-6}$ and $W = 5 \times 10^{-5}$ J (given)
   Therefore, using equation (1), we have
   $W = -q\,(V_2 - V_1)$
   ⇨ $5 \times 10^{-5} = -2 \times 10^{-6}\,(0 - 0.02\,a)$
   ⇨ **$a = 1.25 \times 10^3\ V/m^2$**

2. For any plane parallel to X-Y plane, z = constant
   Hence,
   $V = ax^2 + ay^2 + 2az^2$
   ⇨ $V = ax^2 + ay^2 + \text{constant}$ (because, a and z are constants)
   Also, for equipotential surface, V = constant.
   Hence,
   $V = ax^2 + ay^2 + \text{constant}$
   ⇨ $ax^2 + ay^2 = \text{constant}$ for an equipotential surface

   Since,

   $ax^2 + ay^2 = \text{constant} = k$ (say)

   ⇨ **$ax^2 + ay^2 = k$ which is an equation of a circle.**

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3. Substituting V = 6250 V (given), z = √2 m (given) and a = 1.25 x 10^3 V/m^2 (from part 1) in equation $V = ax^2 + ay^2 + 2az^2$, we get

   $V = ax^2 + ay^2 + 2az^2$

   ⇨ $6250 = 1.25 \times 10^3 (x^2 + y^2 + 2 (\sqrt{2})^2)$

   ⇨ **$x^2 + y^2 = 1$**

   This is an equation of circle of radius 1 m.

Chemistry

4. (a)

   Since, Q forms precipitate with both X and Y, it must be K_2CO_3 because Ag_2CO_3 and $BaCO_3$ are insoluble salts.

5. (b)

   K_2CO_3 on mixing with $Pb(NO_3)_2$ will give precipitate $PbCO_3$ but no precipitate will be formed if K_2CO_3 is combined with NaCl solution. Therefore, R is $Pb(NO_3)_2$ and S is NaCl.

6. (a)

   If X forms a precipitate with S = NaCl but not with R = $Pb(NO_3)_2$, it must be $AgNO_3$. On the other hand if Y forms precipitate only with Q but not with R and S, it must be $BaCl_2$.

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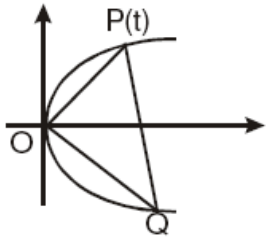
Mathematics

7. (d)

If PQ is a normal at $P(t)$, then the parameter of Q is $t_1 = -t - \frac{2}{t}$



Since $\angle POQ = 90^o, \frac{2}{t} \times \frac{2}{t_1} = -1$ (i.e.) $tt_1 + 4 = 0$ (i.e.) $t\left(-t - \frac{2}{t}\right) + 4 = 0$

$t^2 = 2 \quad \therefore \ t = \sqrt{2}$ taking P above the x-axis and

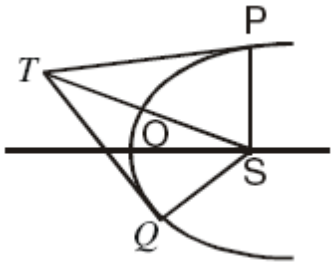
$t_1 = -\frac{4}{t} = -2\sqrt{2}$

Length of $PQ = \sqrt{(at^2 - at_1^2)^2 + (2at - 2at_1)^2} = a\sqrt{(2-8)^2 + (2\sqrt{2} + 4\sqrt{2})^2} = a\sqrt{36+72} = 6\sqrt{3a}$

8. (b)

If $P(t_1), Q(t_2)$, then $T = (t_1t_2, t_1 + t_2), S = (1,0)$ for $y^2 = 4x$

$a = SP = (1 + t_1^2)$ and $c = SQ = (1 + t_2^2)$



Now $b^2 = ST^2 = (t_1t_2 - 1)^2 + (t_1 + t_2)^2$

$= t_1^2 + t_2^2 + 1 + t_1^2t_2^2 = (1 + t_1^2)(1 + t_2^2) \quad \therefore \ b^2 = ac$

$\therefore$ discriminant of the equation $4b^2 - 4ac = 0$

$\therefore$ roots of $ax^2 + 2bx + c = 0$ are real and equal

9. (d)

The chord of contact of tangents from $P(x_1, y_1)$ is $yy_1 - 2ax - 2ax_1 = 0$

Since this touches $x^2 = 4by$, the roots of the quadratic in x

$y_1\frac{x^2}{4b} = 2ax - 2ax_1 = 0$ are equal

(i.e.) $y_1x^2 - 8abx - 8abx_1 = 0$ has equal roots

$$64a^2b^2 + 32aby_1x_1 = 0$$

$\therefore$ locus of (x_1, y_1) is the curve $xy = -2ab$ which is a rectangular hyperbola