DAILY TEST SERIES FOR IIT-JEE 2009 FROM VIDYA DRISHTI

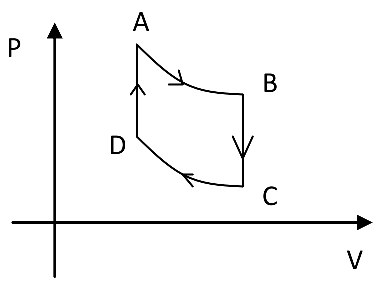
13.03.2009

Total time for three comprehensions: 20 min
Linked comprehension type questions

Physics

Paragraph for comprehension 1 to 3

One mole of a monatomic ideal gas is taken through the cycle shown in figure.



Process A to B: adiabatic expansion
Process B to C: constant volume cooling
Process C to D: adiabatic compression
Process D to A: constant volume heating

The pressure and temperature at A, B etc. are denoted by P_A, T_A, P_B, T_B etc., respectively. Given T_A =1000 K, P_B = (2/3) P_A and P_C = (1/3) P_A. Use $\left(\frac{2}{3}\right)^{\frac{2}{5}} = 0.85$

1. The work done by the gas in the process A to B is
   (a) 1870 J
   (b) 5300 J
   (c) 6200 J
   (d) 8430 J

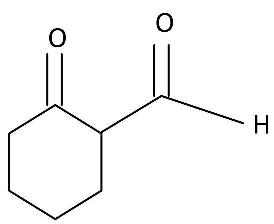
2. The heat lost by the gas in the process A to B is
   (a) 1870 J
   (b) 5300 J
   (c) 6200 J
   (d) 8430 J

3. The temperature T_D is
   (a) 1000 K
   (b) 850 K
   (c) 500 K
   (d) 425 K

Chemistry

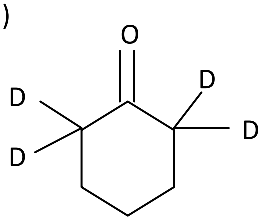
Paragraph for comprehension 4 to 6

Consider the compound shown below

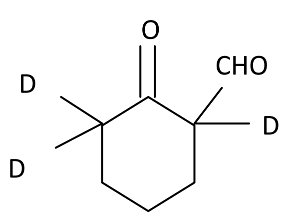


Choose correct option/s.

4. The product of its reaction with excess of D_2O
   (a)



   (b)



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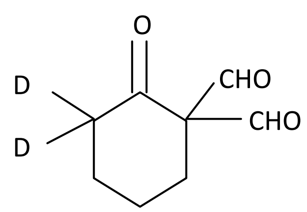
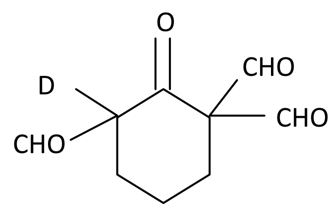
(c)

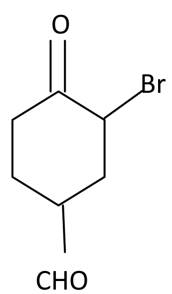


(d)

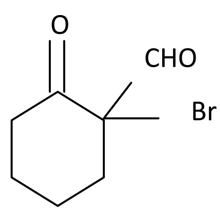


5. The product of its reaction with one mole of Br_2 in dilute acidic medium

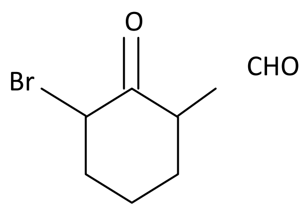
(a)



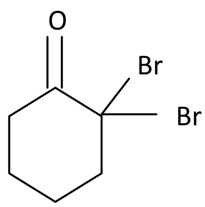
(b)



(c)

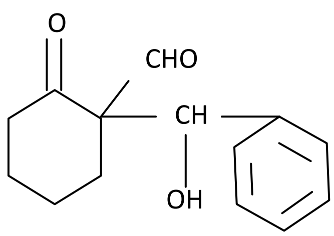


(d)

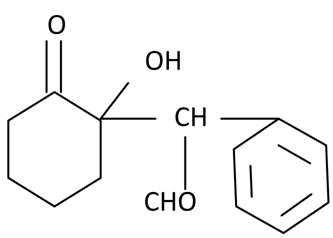


6. The product of its reaction with one mole of C_6H_5CHO in presence of strong base EtONa.

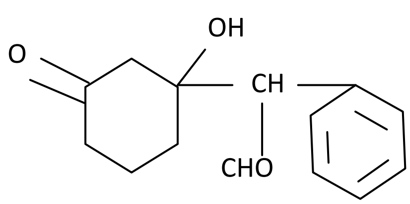
(a)



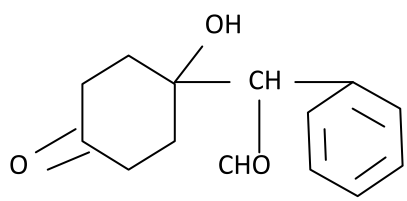
(b)



(c)



(d)



Mathematics

Paragraph for comprehension 7 to 9

Let f (x) = $x^3 - 9x^2 + 15x + 6$ and

g (x) = $\begin{cases} \min f(t), & 0 \le t \le x, \quad 0 \le x \le 6 \\ x - 18, & x > 6 \end{cases}$

7. g (x) is continuous for all x Є
   (a) R
   (b) $(-\infty, 0)$
   (c) $(0, \infty)$
   (d) $[0, \infty)$

8. g (x) is differentiable for all x Є
   (a) $(0, \infty)$
   (b) $(0, \infty) - \left\{\frac{9-\sqrt{21}}{2}\right\}$
   (c) $[0, \infty)$
   (d) $[0, \infty) - \left\{\frac{9-\sqrt{21}}{2}\right\}$

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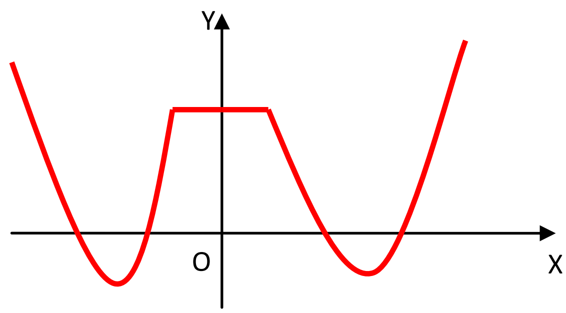
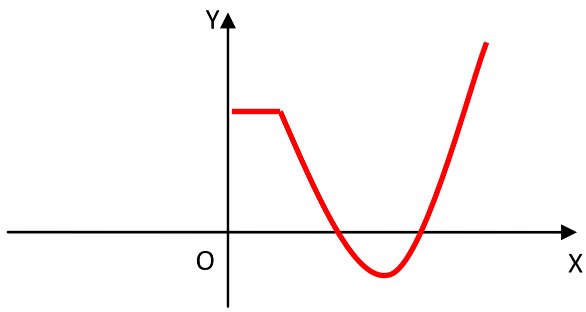
9. Approximate graph of g(x) is

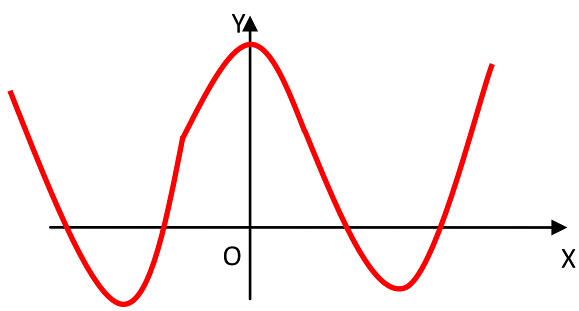
(a)



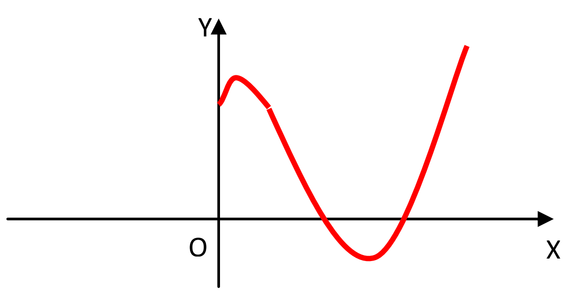
(b)



(c)



(d)



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SOLUTION:

Physics

1. **(a)**
2. **(b)**
3. **(c)**

Given, T_A =1000 K, $P_B = (2/3)\ P_A$ and $P_C = (1/3)\ P_A$.

n = 1 (one mole of gas)

$\gamma = C_p/C_v = 5/3$ (monatomic ideal gas)

1. **Process A to B** is an adiabatic process, therefore,

$$\frac{T^{\gamma}}{P^{\gamma-1}} = \text{constant}$$

$$\Rightarrow \frac{T_A^{\ \gamma}}{P_A^{\ \gamma-1}} = \frac{T_B^{\ \gamma}}{P_B^{\ \gamma-1}}$$

$$\Rightarrow T_B = T_A\left(\frac{P_B}{P_A}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow T_B = 1000\left(\frac{2}{3}\right)^{\frac{5/3-1}{5/3}}$$

$$\Rightarrow T_B = 1000\left(\frac{2}{3}\right)^{\frac{2}{5}}$$

$$\Rightarrow T_B = 1000(0.85) \qquad \left(\text{Given } \left(\frac{2}{3}\right)^{\frac{2}{5}} = 0.85\right)$$

$$\Rightarrow T_B = 850K$$

Therefore, work done in the adiabatic process A to B is

$$W_{12} = \frac{P_2V_2 - P_1V_1}{1-\gamma}$$

$$\therefore\ W_{AB} = \frac{P_BV_B - P_AV_A}{1-\gamma}$$

We need to find out volumes V_A and V_B. Using adiabatic expressions PV^{γ} = constant and $TV^{\gamma-1}$ = constant we can find out V_A and V_B. But it will take too much time. What to do?

Yes! We can use ideal gas equation, PV = nRT.

Hence,

$$\therefore\ W_{AB} = \frac{P_BV_B - P_AV_A}{1-\gamma}$$
$$= \frac{nRT_B - nRT_A}{1-\gamma}$$
$$= \frac{nR(T_B - T_A)}{1-\gamma}$$
$$= \frac{1\times 8.314(850-1000)}{1-\frac{5}{3}}$$
$$\approx 1870 \text{ J}$$

2. **Process B to C** is an isochoric process (V = constant). Therefore,

$$T \propto P$$
$$\Rightarrow \frac{T_B}{T_C} = \frac{P_B}{P_C}$$
$$\Rightarrow T_C = T_B\left(\frac{P_C}{P_B}\right)$$
$$\Rightarrow T_C = 850\left(\frac{(1/3)P_A}{(2/3)P_A}\right)$$
$$\Rightarrow T_C = 425K$$

Now from first law of thermodynamics,

$\Delta U = \Delta Q - \Delta W$

⇨ $nC_v\Delta T = \Delta Q - 0$ (using $\Delta U = n\,C_v\Delta T$ and $\Delta W = 0$ for constant volume process)

⇨ $\Delta Q = nC_v\Delta T$

Therefore,

$Q_{BC} = nC_v(T_C - T_B)$

⇨ $Q_{BC} = nC_v(T_C - T_B)$

⇨ $Q_{BC} = n\,(3/2)\,R\,(T_C - T_B)$ (because, $C_v = (3/2)\,R$ for a monatomic ideal gas)

⇨ $Q_{BC} = 1 \times (3/2) \times 8.314\,(425 - 850)$

⇨ **$Q_{BC} \approx$ 5300 J**

3. This part is bit tricky.

Process C to D is adiabatic. Therefore,

$$\frac{T_C^{\ \gamma}}{P_C^{\ \gamma-1}} = \frac{T_D^{\ \gamma}}{P_D^{\ \gamma-1}}$$

$$\Rightarrow \frac{P_C}{P_D} = \left(\frac{T_D}{T_C}\right)^{\frac{\gamma}{1-\gamma}} \qquad ...(1)$$

Similarly, process A to B is adiabatic. Therefore,

$$\Rightarrow \frac{P_A}{P_B} = \left(\frac{T_B}{T_A}\right)^{\frac{\gamma}{1-\gamma}} \qquad ...(2)$$

Multiplying (1) & (2) we get,

$$\frac{P_C P_A}{P_D P_B} = \left(\frac{T_D T_B}{T_C T_A}\right)^{\frac{\gamma}{1-\gamma}} \qquad ...(3)$$

Again, processes B to C and D to A are isochoric (V =constant), therefore,

$$\frac{P_C}{P_B} = \frac{T_C}{T_B} \quad \& \quad \frac{P_A}{P_D} = \frac{T_A}{T_D}$$

Multiplying these two equations, we get

$$\frac{P_C P_A}{P_B P_D} = \frac{T_A T_C}{T_D T_B} \qquad ...(4)$$

From (3) and (4), we have

$$\left(\frac{T_D T_B}{T_C T_A}\right)^{\frac{\gamma}{1-\gamma}} = \frac{T_A T_C}{T_D T_B}$$

$$\Rightarrow \frac{T_A T_C}{T_D T_B} = 1$$

$$\Rightarrow T_D = \frac{T_A T_C}{T_B}$$

$$\Rightarrow T_D = \frac{1000 \times 425}{850}$$

$$\Rightarrow T_D = 500K$$

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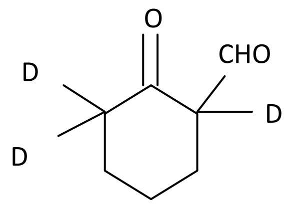
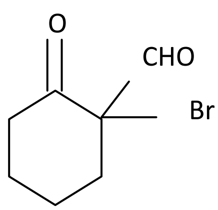
Chemistry

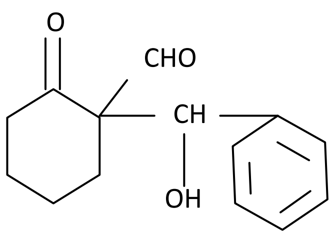
4. (b)



5. (b)



6. (a)



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Mathematics

7. **(d)**
8. **(d)**
9. **(b)**

$$f(x) = x^3 - 9x^2 + 15x + 6$$

$$\therefore f'(x) = 3x^2 - 18x + 15$$

$$f'(x) < 0, x \in (1,5)$$

$$f'(x) > 0, x \in (-\infty,1) \cup (5,\infty)$$

$$f(0) = 6$$

$$\therefore f(x) = 6$$

$$\Rightarrow x^3 - 9x^2 + 15x + 6 = 0$$

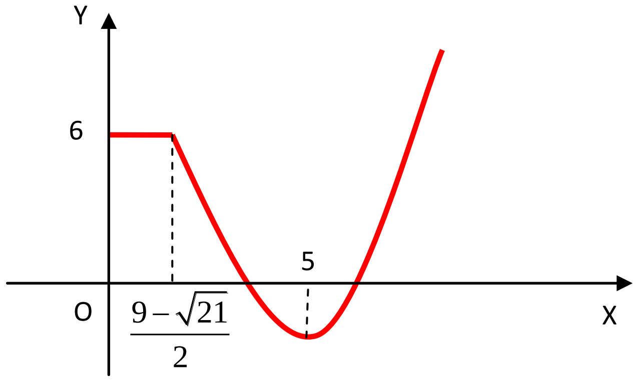
$$\Rightarrow x = 0, x = \frac{9 \pm \sqrt{21}}{2}$$

$$Here, \frac{9+\sqrt{21}}{2} > 6$$

$$\therefore x = \frac{9-\sqrt{21}}{2}$$

$$\therefore g(x) = \begin{cases} 6, & 0 \le x < \dfrac{9-\sqrt{21}}{2} \\ x^3 - 9x^2 + 15x + 6, & \dfrac{9-\sqrt{21}}{2} \le x \le 6 \\ x - 18, & x > 6 \end{cases}$$

Thus graph of g(x) is shown below



It is clear for graph of g(x) that it is continuous on $[0,\infty)$

& differentiable on $[0,\infty) - \frac{9-\sqrt{21}}{2}$