DAILY TEST SERIES FOR IIT-JEE 2009 FROM VIDYA DRISHTI

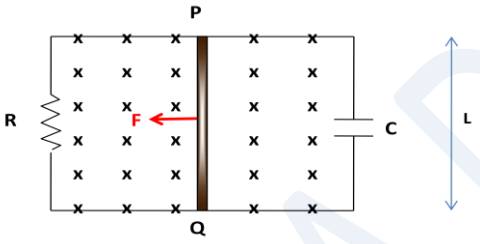
02.04.2009

Total time: 20 min
Single/Multiple choice questions (one or more options may correct)

Physics

1. A 5 henry inductor is placed in series with a 10 ohm resistor. An emf of 5 V is suddenly applied to the combination. Then for time = one time constant
   (a) The rate at which energy is delivered by the battery is 1.58 W
   (b) Power dissipated as heat is 0.998 W
   (c) Power stored in magnetic field is 0.582 W
   (d) Power delivered by battery = power stored in magnetic field + power dissipated as heat

2. A metallic rod of AC of mass1 kg can slide freely on two vertical conducting poles separated by a distance 10 m. a uniform magnetic field B = 0.5 T is present perpendicular to poles inwards. Resistance of the rod between the poles is 5 ohm and a constant emf E = 20 V is applied as shown in figure. The rod is released from rest. Then the speed of the rod is



   (a) directly proportional to time t
   (b) inversely proportional to time t
   (c) depends exponentially on time t
   (d) independent of t

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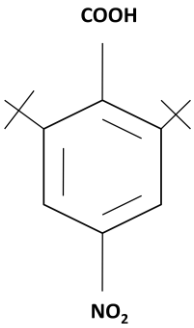
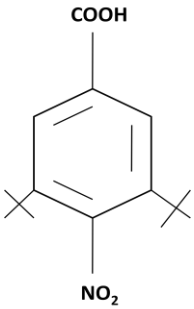
Chemistry

3. Consider two compounds A and B. Then,



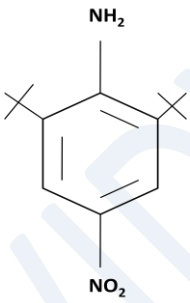
A



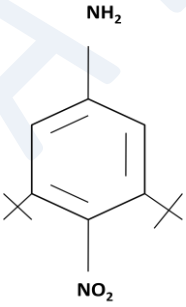
B

(a) A is stronger acid
(b) B is stronger acid
(c) Both are equally acidic
(d) not answerable

4. Consider two compounds A and B. Then,



A



B

a) A is stronger base
b) B is stronger base
c) Both are equally basic
d) not answerable

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Mathematics

5. The value of $|\hat{i}\times\vec{a}|^2+|\hat{j}\times\vec{a}|^2+|\hat{k}\times\vec{a}|^2=$

a) 0

b) $\vec{a}^2$

c) $2\vec{a}^2$

d) $3\vec{a}^2$

6. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2$ does not exceed

a) 4

b) 9

c) 8

d) 6

SOLUTION:

1. **(a), (b), (c), (d)**
2. **(c)**
3. **(a)**
4. **(b)**
5. **(c)**
6. **(b)**

EXPLANATIONS FROM NEXT PAGE

Physics

1. a, b, c, d

At any instant current in an LR circuit is given by

$$i = i_0\left(1 - e^{-Rt/L}\right) = \frac{E}{R}\left(1 - e^{-Rt/L}\right)$$

At t = one time constant = $\frac{L}{R}$

$$i = \frac{E}{R}\left(1 - e^{-1}\right)$$

$$\Rightarrow i = \frac{5}{10}\left(1 - 2.718^{-1}\right)$$

$$\Rightarrow i = 0.316A$$

∴ The *rate at which energy* (power) is delivered by the battery is

$P_1 = Ei = 5x0.316 = 1.58$ W …(1)

The power dissipated as heat in the resistor is given by

$P_2 = i^2R = (0.316)^2 x\ 10 = 0.998$ W …(1)

Power stored in the inductor is given by

$$P_3 = Li\left(\frac{di}{dt}\right)$$

$$\because i = \frac{E}{R}\left(1 - e^{-Rt/L}\right)$$

$$\Rightarrow \frac{di}{dt} = \frac{E}{L}e^{-Rt/L}$$

$$\therefore P_3 = Li\left(\frac{di}{dt}\right) = iEe^{-Rt/L}$$

Therefore, at time t = one time constant = $\frac{L}{R}$

$$P_3 = iEe^{-1}$$

$$\Rightarrow P_3 = iEe^{-1}$$

$$\Rightarrow P_3 = 5 \times 0.316 \times 2.718^{-1}$$

$$\Rightarrow P_3 = 0.582W \quad \ldots(3)$$

From equations (1), (2) & (3) we can see that

$P_1 = P_2 + P_3$

$\Rightarrow$ Power delivered by battery = power stored in magnetic field + power dissipated as heat

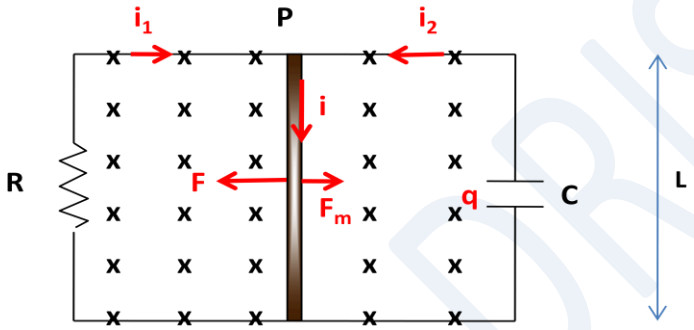
It is same as required by the principle of conservation of energy.

2. (c)

Let v be the velocity of the rod at any time t.

Motional emf E =Bvl

Charge in the capacitor, q = CE = BvlC



Current through the resistor, $i_1 = \frac{E}{R} = \frac{Bvl}{R}$

Current through the capacitor, $i_2 = \frac{dq}{dt} = BlC\frac{dv}{dt}$

Therefore, current through the rod $i = i_1 + i_2 = \frac{Bvl}{R} + BlC\frac{dv}{dt}$

Magnetic force on the rod, $F_m = ilB = B^2l^2\left(\frac{v}{R} + C\frac{dv}{dt}\right)$

Therefore, net force on the rod

$$F_{net} = F - F_m$$

$$\Rightarrow F_{net} = F - B^2l^2\left(\frac{v}{R} + C\frac{dv}{dt}\right)$$

$$\Rightarrow ma = F - B^2l^2\left(\frac{v}{R} + C\frac{dv}{dt}\right)$$

$$\Rightarrow m\frac{dv}{dt} = F - B^2l^2\left(\frac{v}{R} + C\frac{dv}{dt}\right)$$

$$\Rightarrow (m + B^2l^2C)\frac{dv}{dt} = F - B^2l^2\frac{v}{R}$$

$$\Rightarrow \frac{dv}{F - B^2l^2\frac{v}{R}} = \frac{dt}{m + B^2l^2C}$$

$$\Rightarrow \int_0^v \frac{dv}{F - B^2l^2\frac{v}{R}} = \int_0^t \frac{dt}{m + B^2l^2C}$$

$$\Rightarrow v = \frac{FR}{B^2l^2}\left[1 - e^{-\frac{B^2l^2}{R(m+B^2l^2C)}t}\right]$$

Chemistry

3. (a)

A is a stronger acid than B although electron donating tertiary butyl groups are closer to –COOH functional group. It is due to steric inhibition to resonance of $–NO_2$ group by two bulky tertiary butyl groups in B, while $–NO_2$ is free for resonance in A and increases acid strength markedly by –R effect.

4. (b)

B is stronger base than A. Here basic strength is controlled by –R effect of $–NO_2$. In B, due to steric inhibition to resonance of $–NO_2$ by two bulky tertiary butyl group, it is not decreasing basic strength by –R effect, hence B is stronger base than A.

Mathematics

5. (c)

Let $\vec{a}$ make angles α, β and γ with the directions $\hat{i}, \hat{j}$ and $\hat{k}$ so that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Again square of a vector is square of its modulus

$$\therefore |\hat{i}\times\vec{a}|^2 = (\hat{i}\times\vec{a})^2 = a^2\sin^2\alpha$$

$$\therefore \sum|\hat{i}\times\vec{a}|^2 = a^2\sum\sin^2\alpha = a^2\ \ 1-\cos^2\alpha+1-\cos^2\beta+1-\cos^2\gamma$$

$$\sum|\hat{i}\times\vec{a}|^2 = a^2(3-1) = 2a^2 = 2\vec{a}^2$$

6. (b)

$$E = |\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2$$

$$= 2(1+1+1) - 2\sum\vec{a}.\vec{b}$$

$$= 6 - 2\sum\vec{a}.\vec{b}$$

$$\because\ \vec{a}+\vec{b}+\vec{c}^{\ 2} \geq 0$$

$$\therefore (1+1+1) + 2\sum\vec{a}.\vec{b} \geq 0$$

$$\therefore 3 \geq -2\sum\vec{a}.\vec{b}$$

$$\Rightarrow 3 \geq E - 6$$

$$\Rightarrow E \leq 9$$